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# RESEARCH MEMORANDUM

ANALYSIS OF TURBINE STATOR ADJUSTMENT REQUIRED FOR  
COMPRESSOR DESIGN-POINT OPERATION IN HIGH  
MACH NUMBER SUPERSONIC TURBOJET ENGINES

By Robert E. English and Richard H. Cavicchi

Lewis Flight Propulsion Laboratory  
Cleveland, Ohio

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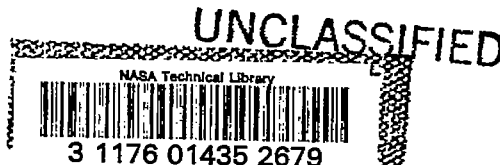
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RESEARCH MEMORANDUMANALYSIS OF TURBINE STATOR ADJUSTMENT REQUIRED FOR COMPRESSOR DESIGN-  
POINT OPERATION IN HIGH MACH NUMBER SUPERSONIC TURBOJET ENGINES

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## SUMMARY

The method of engine off-design operation that keeps the compressor at its design point by means of turbine stator adjustment was evaluated analytically as a means of obtaining high engine thrust during take-off and flight at low Mach numbers from an engine suitable for operation at supersonic speeds. The analysis considers a particular type of turbine design for each of the two following sets of engine design conditions: (1) flight Mach number of 2.5, turbine-inlet temperature of  $2000^{\circ}$  R, and compressor pressure ratio of 3; and (2) flight Mach number of 3.0, turbine-inlet temperature of  $3000^{\circ}$  R, and compressor pressure ratio of 2. Take-off operation at sea level was considered representative of the general class of off-design operating conditions.

For this mode of engine operation, the effect of incorporating turbine stator adjustment is to make the turbine design sensitive to the particular engine design conditions selected. For some engine design conditions, the turbine must be conservatively designed for the high-speed flight conditions in order that the turbine can operate satisfactorily for take-off. For such an engine, the engine thrust at take-off and at flight at low Mach numbers should be limited to the minimum acceptable values in order to reduce the impairment of the engine design-point performance. A new concept, the break-even point, can be used to evaluate quickly whether or not a turbine designed for a given service will approach the blade-loading limit during off-design operation.

## INTRODUCTION

Operation of turbojet engines in airplanes at flight Mach numbers above 2 presents a problem in engine design because of the wide range of ram temperature over which the engine must operate. For example, the ram temperature varies from  $518.4^{\circ}$  R for sea-level take-off conditions to  $883^{\circ}$  R for a flight Mach number of 2.5 and  $1098^{\circ}$  R for a flight Mach number of 3.0 in the stratosphere. These flight values are 171 and 212 percent, respectively, of the take-off value. If an engine is operated at constant engine temperature ratio (ratio of turbine-inlet to compressor-inlet temperature) and constant compressor equivalent rotational speed, the values of all engine equivalent variables (such as compressor



equivalent weight flow, compressor pressure ratio, turbine equivalent rotational speed, and turbine pressure ratio) remain essentially constant. The advantage of this type of operation is that the compressor and turbine continue to operate under conditions closely approximating their design conditions, and good component performance can therefore be anticipated for the entire range of flight conditions. But the disadvantage is that, for engine operation within a specified limit on turbine-inlet temperature during high-speed flight, very low engine thrust is available for both take-off and flight at low Mach number, because the turbine would be operating far below a temperature limit imposed by material properties. For example, a turbine designed for operation with a turbine-inlet temperature of  $2000^{\circ}\text{R}$  and a flight Mach number of 2.5 would have a turbine-inlet temperature of only  $1174^{\circ}\text{R}$  for take-off. In the same way, a turbine-inlet temperature of  $3000^{\circ}\text{R}$  during flight at a Mach number of 3.0 would be reduced to  $1415^{\circ}\text{R}$  during take-off.

A way to circumvent this operational difficulty has previously been considered at the NACA Lewis laboratory. It was suggested that the engine be designed for the flight condition in order to achieve satisfactory operation in flight and that for operation under other than the design conditions, the compressor equivalent rotational speed and compressor pressure ratio be maintained constant. High turbine-inlet temperature and, concomitantly, high engine thrust at low flight Mach number could be obtained by selecting a control that would adjust the angle and throat area of the turbine-entrance stator in order to maintain constant compressor pressure ratio and simultaneously adjust the exhaust-nozzle throat area to maintain constant compressor equivalent rotational speed. In this way, the Mach numbers within the compressor would not be raised above the design values, nor would the compressor pressure ratio be increased. For these reasons, the compressor should operate for take-off essentially as designed.

On the other hand, this method of engine operation requires that the turbine be capable of handling a wide range of turbine-entrance equivalent weight flow and equivalent rotational speed. For example, a turbine intended for flight at a Mach number of 3.0 must operate during take-off with an equivalent rotational speed less than 69 percent and an entrance equivalent weight flow greater than 145 percent of the flight, or design, values if a constant turbine-inlet temperature is to be maintained. Whether or not the turbine is capable of satisfactory operation when subjected to such a range of variation from design is a problem that requires a study of flow conditions within the turbine for the particular application being considered.

For this method of engine operation, the range of conditions over which the turbine must operate requires that the process of turbine design consider a design range rather than a design point. In practice, that operating condition for which engine performance is most critically sensitive to turbine performance will be most seriously considered in the turbine design; the other operating conditions in the range of design will generally only restrict or qualify the turbine design. If the designer

desires that the engine performance be superlative during high-speed flight and if he is willing to accept reasonable penalties in turbine efficiency under other operating conditions in order to realize this superlative high-speed performance, the high-speed flight conditions will receive the principal emphasis in design; this point of view has been adopted in this report. In order to avoid circumlocution, this high-speed flight condition is herein referred to as the "design point" and other operating conditions as "off-design."

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An investigation of the variation in turbine-entrance equivalent weight flow for a two-stage turbine equipped with an adjustable turbine stator is presented in reference 1. For this investigation, the stator area was varied from 88.5 to 128.4 percent of the design area, or a total variation of 45 percent of the smallest area. For this 45-percent increase in area, the turbine-entrance equivalent weight flow was increased only 10 percent. After this rise in flow was obtained, a further increase in stator flow area produced no additional rise in turbine-entrance equivalent flow. This particular turbine is incapable of realizing the desired 45-percent rise in turbine-entrance equivalent weight flow. Because this turbine was not specifically designed for stator adjustment but was intended instead for investigation of compressor characteristics during engine operation, its performance is probably not typical of the performance of adjustable-stator turbines. On the other hand, the characteristics of this turbine serve to emphasize some of the turbine problems associated with turbine stator adjustment for design-point operation of compressors.

A general analytical investigation of turbine stator adjustment is presented in reference 2. In figure 1 of reference 2 is shown the manner in which the stator-exit angle is expected to vary with turbine-entrance temperature if the turbine stator is assumed to be choked over the entire operating range and if the compressor-entrance conditions are constant. The abscissa, ratio of turbine-entrance temperature to the design value, is proportional to the square of the turbine-entrance equivalent weight flow. In this figure, the turbine-entrance equivalent weight flow rises continuously with rising values of stator-exit angle. The departure of the turbine in reference 1 from this trend must, therefore, be due to lack of choking in the turbine stator.

In reference 2, the effect of stator adjustment on turbine off-design performance is also investigated by analytically computing the performance of a particular single-stage turbine. The turbine-entrance equivalent weight flow was varied 24 percent, which, although higher than the 10-percent variation in reference 1, is still considerably below the 45 percent required for flight at a Mach number of 3.0. Apparently, the attainable variation in turbine-entrance equivalent weight flow depends on the particular turbine design, and the possibility of obtaining a 45-percent increase in turbine-entrance equivalent weight flow has not yet been established.

As a subordinate part of reference 3, the possible variation in turbine-entrance equivalent weight flow for compressor operation at its

design point was investigated. The analysis of reference 3 is based on an assumption of isentropic flow. If the stator-rotor area ratio is changed from 0.5 to 1.0, figure 14 of reference 3 shows that the turbine-entrance equivalent weight flow can be increased by about 70 percent; of this 70-percent increase, as little as 21 percent is in the region of stator choking, and the remaining 49 percent in the nonchoking region. The trend of computed turbine-entrance equivalent weight flow with increasing turbine stator area indicates that even greater increases can be obtained. Reference 3 thus shows that, unless the turbine losses rise considerably with stator adjustment, large variations in turbine-entrance equivalent weight flow may be anticipated from at least some turbine designs, even in the nonchoking range of design.

For engines suitable for operation over a range of flight conditions from take-off at sea level to flight in the stratosphere at a Mach number of 2.5 or 3.0, this report considers engine off-design operation to be such that, by means of turbine stator adjustment, the compressor is operated at its design pressure ratio and equivalent rotational speed for the entire range of flight conditions. The effect of such operation on the turbine was analyzed at the NACA Lewis laboratory. The purposes of this analysis are: (1) to find the variations to be expected in internal flow conditions of such turbines as an indication of the possible variations in loss within the turbines, and (2) to determine the compromises that must be made in turbine design because of the way in which the engine is operated. In order to investigate good and bad ranges of turbine design, the relation between the turbine design conditions and the engine performance was studied with respect to limits imposed by turbine aerodynamics, centrifugal stress, and turbine-tip frontal area.

In studying the problem of using turbine stator adjustment, a particular type of turbine design was considered for each of two sets of flight, or design, condition:

Flight Mach number	Turbine-inlet temperature, OR	Compressor pressure ratio
2.5	2000	3
3.0	3000	2

Take-off at sea level was the engine off-design operating condition for which engine operation and turbine design were analyzed. Take-off was considered as representative of the general class of off-design operating conditions.

#### SYMBOLS

The following symbols are used in this report. The location of the numerical stations is graphically illustrated in figure 1, along with a presentation of a typical velocity diagram.

A area, sq ft  
a velocity of sound,  $\sqrt{kgRT}$ , ft/sec  
 $a'_{cr}$  critical velocity relative to stator,  $\sqrt{\frac{2k}{k+1} gRT'}$ , ft/sec  
b fraction of mass flow bled from main engine flow  
 $c_p$  specific heat at constant pressure, Btu/(lb)(°R)  
E turbine work, Btu/lb  
F engine thrust, lb  
f fuel-air ratio at exit of primary burner  
g standard acceleration due to gravity, 32.17 ft/sec<sup>2</sup>  
h specific enthalpy, Btu/lb  
J mechanical equivalent of heat, 778.156 ft-lb/Btu  
k ratio of specific heats  
n exponent of polytropic expansion in turbines  
p pressure, lb/sq ft  
R gas constant, 53.4 ft-lb/(lb)(°R)  
r radius, ft  
T temperature, °R  
U blade speed, ft/sec  
V absolute velocity, ft/sec  
W relative velocity, ft/sec  
w weight flow, lb/sec  
 $\alpha$  angle of absolute flow measured from direction of blade motion, deg  
 $\beta$  angle of relative flow measured from direction of blade motion, deg  
 $\Gamma$  density of blade metal, lb/cu ft  
 $\delta$  pressure-reduction ratio,  $p/2116$   
 $\eta_{\infty}$  polytropic efficiency for expansion in turbines,  $(k/k-1)(n-1/n)$   
 $\theta$  temperature-reduction ratio,  $T/518.4$

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$\rho$  gas density, lb/cu ft  
 $\sigma$  blading stress, lb/sq ft  
 $\psi$  taper factor  
 $\omega$  angular velocity, radians/sec

## Subscripts:

b break-even condition  
cr critical or choking  
des design  
eq equivalent operating condition  
h hub  
m mean  
t tip  
u tangential component (positive in direction of blade speed U)  
x axial component  
0 free-stream conditions  
1 compressor entrance  
2 compressor exit  
3 turbine entrance  
4 turbine stator exit  
5 turbine rotor exit  
6 exhaust-nozzle exit

## Superscript:

' total or stagnation state

## ANALYSIS AND DISCUSSION

## Limiting Blade Loading and Break-Even Condition

A condition called "limiting blade loading", which has previously been recognized as a problem in turbine aerodynamics (see ref. 4), represents a fundamental limit on the design and operation of turbines. Limiting blade loading affects turbine operation in the following way: If, for any given set of turbine-inlet conditions and turbine rotational speed, the pressure ratio across a single-stage turbine is gradually increased, a condition is eventually reached beyond which any further increase in turbine pressure ratio produces no further rise in turbine torque. This unchanging value of torque indicates that the forces on the rotor blades have reached a maximum; that is, the rotor blades are operating at their loading limit.

This condition of limiting blade loading was shown in reference 5 to prevail if the axial Mach number at the turbine exit is approximately 0.7; and, for safe design, the value of 0.7 should not be exceeded. For a relatively low value of exit tangential Mach number, the equivalent weight flow at the turbine exit  $w_5 \sqrt{\theta_5} / \delta_5$  is directly related to the exit axial Mach number. The limiting value of exit axial Mach number of 0.7 corresponds to an exit equivalent weight flow about 10 percent less than the choking, or critical, value. The critical area  $A_{cr,5}$  is thus a direct measure of the minimum value of exit annular area within the blade-loading limit.

The manner in which the specified mode of engine operation affects the turbine-exit equivalent flow and thus the proximity of the turbine operating condition to the blade-loading limit can be investigated in the following way: Continuity of mass flow requires that

$$(1-b)(1+f) \frac{w_1 \sqrt{T_1'}}{p_1'} = \frac{w_5 \sqrt{T_5'}}{p_5'} \frac{p_2'}{p_1'} \frac{p_3'}{p_2'} \frac{p_5'}{p_3'} \sqrt{\frac{T_1'}{T_3'} \frac{T_3'}{T_5'}} \quad (1)$$

For the specific type of engine operation, that is, with constant compressor pressure ratio and equivalent rotational speed, the compressor equivalent weight flow is also constant. With reasonable accuracy, the fraction of mass flow  $b$  bled from the main engine flow, the fuel-air ratio  $f$ , and the burner pressure ratio  $p_3'/p_2'$  may be assumed to be constant. For expansion along a polytropic path,

$$\frac{p_5'}{p_3'} = \left( \frac{T_5'}{T_3'} \right)^{\frac{n}{n-1}}$$



For the assigned mode of engine operation, the compressor equivalent work is constant, and

$$\left. \begin{aligned} \frac{T_2' - T_1'}{T_1'} &= \text{a constant} \\ \text{or} \\ \frac{T_3' - T_5'}{T_1'} &= \text{a constant} \end{aligned} \right\} \quad (2)$$

Under these conditions, equation (1) reduces to

$$\frac{w_5 \sqrt{T_5'}}{p_5'} \left( \frac{T_5'}{T_3'} \right)^{\frac{n+1}{2(n-1)}} \sqrt{1 - \frac{T_5'}{T_3'}} = \text{a constant} \quad (3)$$

Differentiating equation (3) and setting

$$\frac{d \left( \frac{w_5 \sqrt{T_5'}}{p_5'} \right)}{d(T_5'/T_3')} = 0 \quad (4)$$

yield

$$\left( \frac{T_5'}{T_3'} \right)_b = \frac{n+1}{2n} \quad (5)$$

where the subscript *b* designates the condition for which equation (4) holds and which is herein called "break-even" condition.

Combining equations (2), (3), and (5) gives

$$\frac{\frac{w_5 \sqrt{T_5'}}{p_5'}}{\left( \frac{w_5 \sqrt{T_5'}}{p_5'} \right)_b} = \left[ \frac{\frac{n+1}{2n}}{1 - \left( \frac{n-1}{2n} \right) \frac{(T_3'/T_1')_b}{T_3'/T_1'}} \right]^{\frac{n+1}{2(n-1)}} \sqrt{\frac{T_3'/T_1'}{(T_3'/T_1')_b}} \quad (6)$$

Equation (6) is plotted in figure 2 for two values of  $n$ . The value of  $(w_5 \sqrt{T_5^i}/p_5^i)/(w_5 \sqrt{T_5^i}/p_5^i)_b$  is seen to be a minimum at the break-even point. In the immediate vicinity of this break-even point, small changes in engine temperature ratio move the turbine neither toward nor away from limiting blade loading, hence the name "break-even point". Departure from this break-even point to either higher or lower values of engine temperature ratio increases the turbine-exit equivalent weight flow and moves the turbine towards limiting blade loading. If

$$\frac{T_3^i/T_1^i}{(T_3^i/T_1^i)_b} \rightarrow \frac{n-1}{2n}$$

or

$$\frac{T_3^i/T_1^i}{(T_3^i/T_1^i)_b} \rightarrow \infty$$

then

$$\frac{w_5 \sqrt{T_5^i}}{p_5^i} / \left( \frac{w_5 \sqrt{T_5^i}}{p_5^i} \right)_b \rightarrow \infty$$

For any particular engine, the break-even point may be identified in the following way: From

$$E = c_p (T_3^i - T_5^i)$$

and equation (5),

$$T_{3,b}^i = \frac{2JE}{\eta_\infty R} = \frac{29.1E}{\eta_\infty} \text{ (in } ^\circ\text{R)} \quad (7)$$

For a turbine polytropic efficiency  $\eta_\infty$  of 0.85, equation (7) reduces to

$$T_{3,b}^i = 34.3E \text{ (in } ^\circ\text{R)} \quad (8)$$

For any given turbine work, such as would be required to drive a given compressor at a specified point on the compressor map, equations (7) and (8) permit the break-even temperature  $T_{3,b}^i$  to be determined. For  $k = 1.30$  and  $\eta_\infty = 0.85$ ,  $n$  is 1.244; equation (5) then reduces to

$$\left( \frac{p_3^i}{p_5^i} \right)_b = 1.69 \quad (9)$$

The relation of any particular turbine design to the break-even point can be quickly determined by comparing the turbine pressure ratio  $p_3'/p_5'$  with the break-even value  $(p_3'/p_5')_b$  in equation (9) or by comparing the turbine-inlet temperature  $T_3'$  with the break-even value  $T_{3,b}'$  in equation (7) or (8). Those turbines having pressure ratios greater than 1.69 and turbine-inlet temperatures less than  $T_{3,b}'$  are operating to the left of the break-even point in figure 2; increasing the ratio of turbine-inlet temperature to compressor-inlet temperature and decreasing the turbine pressure ratio moves the operating condition to the right on figure 2.

Engines designed for supersonic flight must be capable of operation at engine temperature ratios higher than those encountered during flight at maximum supersonic Mach number. If for the flight, or design, condition, the turbine pressure ratio  $p_3'/p_5'$  is less than 1.69, the engine temperature ratio is greater than the break-even temperature ratio. Operation of such an engine at low flight Mach numbers or for take-off at high engine temperature ratios will increase the turbine-exit equivalent weight flow above the design value and move the turbine toward limiting blade loading. If the turbine is to be designed so that for one operating condition it operates at the blade-loading limit, such a change in turbine-exit equivalent weight flow requires that a conservative value of turbine-exit equivalent weight flow be selected for the design condition in order that the off-design operational requirements can be fulfilled. This compromising of the design-point performance should be considered in selecting the method of engine off-design operation.

On the other hand, a turbine pressure ratio greater than 1.69 corresponds to an engine temperature ratio less than the temperature ratio at the break-even point. Increasing the engine temperature ratio during low Mach number flight results in moving the turbine away from limiting blade loading until the break-even point is reached. The turbine design does not then need to be compromised to satisfy the off-design requirements, unless the increase in the engine temperature ratio is so great that the break-even point is passed and the value of turbine-exit equivalent weight flow at the highest engine temperature ratios rises above the design value.

Figure 2 shows as well that engines designed for take-off at sea level approach limiting blade loading during operation at constant equivalent rotational speed and reduced engine temperature ratios if at the design point the engine temperature ratio is less than the break-even value; that is, if the turbine pressure ratio at the design point is greater than 1.69. For an engine of this type, which is to incorporate a given compressor, limiting blade loading can be avoided by selecting the turbine-exit annular area to satisfy the requirements of operation

at the lowest engine temperature ratio and accepting the penalty in blade centrifugal stress and turbine-tip frontal area associated with this large annular area.

### Engine Performance

In order to relate the design-point and off-design-point engine performance to the turbine design requirements, the take-off thrust of two possible turbojet engine designs was computed for various take-off operating conditions and the magnitudes of several turbine design parameters were determined. Because no perfectly general way was found to relate these factors, two specific design conditions were selected and the off-design operation restricted to take-off. In both cases the compressor was operated at its design point for take-off.

Engine design condition. - The two engines were considered to be designed for the following conditions:

Flight Mach number	Turbine-inlet temperature, $T_{3,des}^t$ , °R	Compressor pressure ratio, $(p_2'/p_1)_{des}$
2.5	2000	3
3.0	3000	2

The small-stage, or polytropic, efficiencies of both the compressor and turbine were assumed to be 0.85. A burner pressure drop of 0.05 was used. The effect of variation in fuel addition or bleed on mass flow was neglected, that is,  $(1+f)(1-b)$  was assumed to be constant. Between the atmosphere and the compressor inlet, the total-pressure ratio was assumed to be unity. This value of unity was arbitrarily selected and is, of course, not realistic. If a different value were employed, none of the computed results would be affected with the single exception of the thrust ratio  $F/F_{eq}$ . Although the numerical values of  $F/F_{eq}$  would be changed by assumption of a value less than unity for  $p_1'/p_0$ , neither the trends of the results nor the conclusions would be altered thereby.

For these assigned design conditions, the turbine pressure ratios  $(p_3'/p_5')_{des}$  are 2.42 and 1.51 for the Mach 2.5 and the Mach 3.0 engines, respectively. For the Mach 2.5 engine, the design value of turbine pressure ratio is greater than the break-even value of 1.69, and the design condition therefore lies to the left of the break-even point in figure 2.

As will be considered for take-off, increasing the engine temperature ratio above the design value will move the operating condition of the Mach 2.5 engine away from limiting blade loading and toward the break-even point. For the Mach 3.0 engine, the design value of turbine pressure ratio is less than the break-even value. Because the design condition for this turbine lies to the right of the break-even point in figure 2, increasing the engine temperature ratio will cause the turbine to move away from the break-even point in figure 2 and toward the condition of limiting blade loading.

Engine thrust. - For the specified way to operate the engine off-design, the following compressor variables were assumed to have the same values for take-off as for flight under the specified design conditions: pressure ratio  $p_2'/p_1'$ , engine equivalent weight flow  $w_1 \sqrt{\theta_1'}/\delta_1'$ , temperature ratio  $T_2'/T_1'$ , and equivalent blade speed  $U/\sqrt{\theta_1'}$ . Engine and turbine operation were analyzed for two modes of engine off-design operation that satisfy the above restrictions: (1) The turbine-inlet temperature was varied and the turbine stator was adjusted. (2) The engine temperature ratio  $T_3'/T_1'$  was maintained at the design value and the turbine stator was not changed from the design setting. Operating condition (2) is herein referred to as the "equivalent operating condition," because not only are the compressor equivalent parameters kept at their design values but also the turbine pressure ratio and equivalent blade speed are unchanged from design-point operation. The take-off values of turbine-inlet temperature at the equivalent operating conditions are  $1174^\circ$  and  $1415^\circ$  R for the Mach 2.5 and Mach 3.0 engines, respectively. Under these restrictions,

$$\frac{F}{F_{eq}} = \sqrt{\frac{T_5'}{T_{5,eq}'}} \frac{1 - \left(\frac{p_0}{p_5'}\right)^{\frac{k-1}{k}}}{1 - \left(\frac{p_0}{p_5'}\right)_{eq}^{\frac{k-1}{k}}} \quad (10)$$

In figure 3, engine thrust is seen to rise continually as take-off turbine-inlet temperature is raised, until at the design value of turbine-inlet temperature the thrust for the Mach 2.5 engine is 244 percent of the value at the equivalent operating condition, and 206 percent for the Mach 3.0 engine. These curves show the great effect of turbine-inlet temperature on engine thrust, both engines exhibiting essentially the same trend.

Turbine-exit critical area  $A_{cr,5}$ . - The required value of exit annular area is of considerable importance, because, as shown in equation (8) of reference 6,

$$\sigma = \frac{\Gamma \omega^2 r_t^2}{2g} \left[ 1 - \left( \frac{r_h}{r_t} \right)^2 \right] \psi$$

This expression can be modified to read

$$\sigma = \frac{\Gamma U_h^2 \psi}{2g} \left[ \left( \frac{r_t}{r_h} \right)^2 - 1 \right] \quad (11)$$

In order to produce a given amount of turbine work, a certain minimum blade hub speed is required. For specified values of centrifugal stress  $\sigma$ , blade hub speed  $U_h$ , and taper factor  $\psi$ , the particular value of hub-tip radius ratio may be determined from equation (11). For this particular value of hub-tip radius ratio, the turbine-tip frontal area is directly proportional to the annular area. For this reason, if the requirements of off-design operation dictate that the turbine annular area be increased above the value required for satisfactory design-point operation, the engine thrust per unit turbine-tip frontal area for design-point operation is thereby diminished in inverse proportion to the required value of exit annular area. At the blade-loading limit, the critical area  $A_{cr,5}$  is a direct measure of the exit annular area and thus of the frontal area.

Figure 4 shows the manner in which the critical area  $A_{cr,5}$  varies with take-off turbine-inlet temperature for each of the two engines. For the Mach 2.5 engine, the critical area gradually diminishes from the equivalent, or design-point, value to about 0.92 the design-point value. Design-point operation of this engine therefore requires a larger exit annular area than does take-off operation with stator adjustment. If, on the other hand, the engine were to be operated at a thrust level below the thrust for the equivalent operating condition, the engine should not be operated according to the method described herein; the engine equivalent rotational speed should be permitted to fall below the design value. Otherwise, the critical area would exceed the design value and require compromising the engine design-point performance.

For the Mach 3.0 engine, the trend is reversed from that of the Mach 2.5 engine; the design point corresponds to a point to the right of the break-even point in figure 2, with the result that raising the turbine-inlet temperature increases the exit critical area  $A_{cr,5}$  by as

much as 19 percent. Because this rise in critical area results in impairment of the engine design-point performance, it is necessary to keep this impairment to a minimum. One way to accomplish this goal is to require of the engine the minimum thrust that will provide satisfactory off-design operation.

A comparison of the characteristics of these two engines indicates the sensitivity of the turbine design requirements to variation in flight conditions in this range of compressor pressure ratio and turbine-inlet temperature.

Thrust per unit turbine-tip frontal area. - For the Mach 3.0 engine, raising the turbine-inlet temperature above the value of 1415° R for the equivalent operating condition results in a simultaneous increase in both engine thrust and turbine-tip frontal area. Increasing engine size or number of engines of a given size without changing turbine-inlet temperature above the value at the equivalent operating condition also raises the engine thrust and engine frontal area. Which of these two procedures results in the smaller engine for a given required take-off thrust is not immediately apparent. For any given level of centrifugal stress in the turbine blade, engine thrust per unit turbine-tip frontal area is proportional to thrust per unit of critical area  $F/A_{cr,5}$ . For this purpose,

the relation between the thrust-area factor  $\frac{F}{F_{eq}} \frac{A_{cr,5,eq}}{A_{cr,5}}$  and turbine-

inlet temperature for take-off is shown in figure 5 for the Mach 3.0 engine. The engine thrust for a given stress is seen to rise over the entire range of turbine-inlet temperature from 1415° to 3000° R. For this engine, greater take-off thrust is obtained per unit turbine-tip frontal area if the turbine-inlet temperature is increased rather than if the engine size or number of engines is increased.

### Turbine Velocity Diagrams

The turbine velocity diagrams were analyzed for two specific sets of turbine design conditions, because no general way was found to relate engine thrust and the turbine velocity diagrams. The effect of the specified mode of off-design operation on the turbine velocity diagrams was examined for two characteristics: (1) the variation in velocity diagram variables between the design point and various take-off conditions, and (2) the changes required in turbine design for satisfactory turbine operation during take-off. The method by which the velocity diagrams were analyzed is presented in appendix A.

Turbine design conditions. - The turbine work capacity was assumed to be specified by the following relation:

$$\frac{gJE}{U_h^2} = 2.3 \quad (12)$$

From the charts of reference 7, this value was found to be obtainable from a wide range of reaction-limited turbine designs having a very small amount of exit whirl. Hub-tip radius ratios of 0.65 for the Mach 2.5 engine and 0.60 for the Mach 3.0 engine were selected, because they yield good design-point performance for each engine. A limiting value of turbine-exit equivalent weight flow per unit annular area was specified that is 10.5 percent less than the maximum choking value; such an assumption results in an exit axial Mach number no greater than 0.70 for values of turbine-exit tangential Mach number less than about 0.20. This value was used for either design-point or take-off operation, whichever is more severe in this respect. At the design point, the exit tangential velocity was assumed to be zero. The variation in turbine velocity diagram at the mean radius was then determined. The effect of assigning an exit axial Mach number of 0.70 instead of a limiting value of turbine-exit equivalent weight flow per unit annular area is investigated in appendix B.

Engine equivalent weight flow at design point. - The engine equivalent weight flow that such a turbine could pass per unit tip frontal area is computed and plotted in figure 6. The specified mode of engine operation is shown not to affect the engine equivalent weight flow for the Mach 2.5 engine but to decrease the engine equivalent weight flow for turbines of a given size by 16 percent for the Mach 3.0 engine; these values are a direct reflection of the effect of limiting blade loading on exit critical area  $A_{cr,5}$ .

Stator-exit conditions. - The manner in which this type of operation affects the stator-exit flow condition is shown in figures 7 to 9. For the Mach 2.5 engine the maximum adjustment of the stator-exit flow angle  $\alpha_4$  is  $11^\circ$ . The  $\sin \alpha_4$ , which is proportional to the stator-exit flow area, must be increased 42 percent above the design-point value in order that the Mach 2.5 engine can be operated at rated turbine-inlet temperature for take-off; this 42-percent change results from the combination of a 30-percent change in turbine-entrance equivalent weight flow and a 9-percent change in equivalent weight flow per unit stator-exit area because of the reduction in stator-exit Mach number  $(V/a)_4$ . The stator-exit Mach number diminishes from 1.0 to 0.72 for take-off operation at rated turbine-inlet temperature.

Several characteristics of the Mach 3.0 engine are in marked contrast to those of the Mach 2.5 engine, one of which is the continual change in turbine design conditions as the turbine-inlet temperature for take-off is increased. In figure 7(b), for example, this characteristic is exhibited



as a change in stator-exit flow angle  $\alpha_4$  with the take-off value of turbine-inlet temperature. This change in design-point conditions results from limits imposed by the blade-loading limit. Another contrasting feature is that for take-off operation at rated turbine-inlet temperature, the stator-exit flow angle  $\alpha_4$  must be changed by  $58^\circ$ , from  $34^\circ$  to  $92^\circ$ . This turning of the stator-exit flow past the axial direction (stator-exit angle greater than  $90^\circ$ ) is required in order to keep the rotor-entrance tangential velocity  $V_{u,4}$  sufficiently low (even negative in this case) in order to produce no more than the required work. The  $\sin \alpha_4$ , and thus the stator throat area, must be increased by 79 percent for take-off operation at  $3000^\circ \text{ R}$ . Under these conditions, the stator-exit Mach number  $(V/a)_4$  is 0.72 at the design operating condition and 0.52 with the take-off turbine-inlet temperature of  $3000^\circ \text{ R}$ . Despite the leveling off of the  $\sin \alpha_4$ , and hence of the stator-exit flow area for take-off shown in figure 8(b), the ratio of the take-off value of turbine-entrance equivalent weight flow to the design value continues to rise in direct proportion to the square root of the turbine-inlet temperature. Furthermore, this trend would continue even if the turbine-inlet temperature were raised above  $3000^\circ \text{ R}$ , despite the fact that in this temperature range the  $\sin \alpha_4$ , and thus the stator-exit flow area for take-off are decreasing. This trend can continue even though the stator-exit flow area for take-off decreases, because the design value of stator-exit area is decreasing and the stator-exit Mach number for take-off is rising.

Rotor-entrance conditions. - The variation in rotor-entrance conditions with stator adjustment is illustrated in figures 10 and 11. For the Mach 2.5 engine, as the turbine-inlet temperature for take-off increases, the rotor-entrance Mach number  $(W/a)_4$  decreases, the rotor-entrance angle  $\beta_4$  increases (that is, the rotor turning decreases), and the angle of incidence at the leading edge of the rotor becomes negative. On the other hand, the turbine for the Mach 3.0 engine has decidedly different characteristics. The rotor-entrance Mach number  $(W/a)_4$  for the design operating condition decreases as the turbine-inlet temperature for take-off is increased. For values of turbine-inlet temperature from  $1415^\circ$  to  $1900^\circ \text{ R}$ , the rotor-entrance Mach number has practically no variation between the design and take-off operating conditions; the turbine-inlet temperature must be increased to  $2500^\circ \text{ R}$  for take-off before the Mach number gets as high as for an engine not equipped for stator adjustment. For a turbine-inlet temperature of  $3000^\circ \text{ R}$ , the rotor-entrance Mach number for take-off is 42 percent higher than for the design operating condition. The trend of rotor-entrance angle  $\beta_4$  with turbine-inlet temperature is in the same direction for both the Mach 3.0 and the Mach 2.5

engines, but the change is more extreme for the Mach 3.0 engine; for take-off operation with 3000° R, the rotor-entrance angle  $\beta_4$  is 51° higher than for operation at the design point.

Rotor-exit conditions. - The manner in which the rotor-exit conditions varied with turbine-inlet temperature for take-off is presented in figures 12 to 14. For the Mach 2.5 engine, rotor-exit Mach number  $(W/a)_5$  is less for take-off operation than for the design operating conditions. This, from the point of view of Mach number level, is a conservative trend. The velocity ratio  $W_5/W_4$  is less for take-off with a turbine-inlet temperature between 1180° and 1750° R than for the design operating conditions and greater for turbine-inlet temperatures between 1750° and 2000° R. The adverse trend of velocity ratio  $W_5/W_4$  in the temperature range of 1180° to 1750° R will require more conservative turbine design than would be the case if turbine stator adjustment were not used for engine take-off operation with turbine-inlet temperatures between 1180° and 1750° R. The leaving loss  $V_{u,5}^2/2gJE$  is subject to only small variation over the entire range of turbine-inlet temperature ratio.

For the Mach 3.0 engine, the rotor-exit Mach number  $(W/a)_5$  rises for take-off values of turbine-inlet temperature above that for the design operating condition to a value of 1.2 with a turbine-inlet temperature of 3000° R. The velocity ratio  $W_5/W_4$  has a conservative trend throughout this entire range. The leaving loss rises so that, at high turbine-inlet temperatures, it reaches prohibitively high values.

#### Critical Comment

An examination of the trends of engine thrust and turbine design variables with turbine-inlet temperature indicates that, although large increases in take-off thrust can be obtained by operating engines with the compressor at its design point and the turbine-inlet temperature near its rated value, relatively severe adjustments in the turbine internal flow conditions are required that may significantly impair the turbine performance and result in obtaining actual performance considerably inferior to that presented herein. The off-design performance of the Mach 3.0 engine may suffer considerably, because at high turbine-inlet temperatures the stator setting, rotor-entrance angle, and exit tangential velocity all deviate considerably from the design values; in addition, the rotor-entrance Mach number rises considerably above the design value except for low values of turbine-inlet temperature. These deviations in flow conditions from the design flow conditions are great enough that at high take-off values of turbine-inlet temperature the turbine may be

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incapable of passing the required equivalent weight flow. For the Mach 3.0 engine with a turbine-inlet temperature for take-off of 3000° R, operation in the specified way requires rather severe compromises of the turbine design-point performance in order that the blade-loading limit will not be exceeded during off-design operation. In order to minimize the compromising of the design condition, the off-design engine thrust should be limited to the minimum acceptable value.

For the Mach 2.5 engine, the changes in turbine internal flow conditions are much less severe than for the Mach 3.0 engine with the single exception of the velocity ratio  $W_5/W_4$ . At low take-off values of turbine-inlet temperature, the value of velocity ratio is less than at the design point. This condition can be remedied by raising the turbine-inlet temperature closer to the rated value, and thereby the engine thrust will very likely be increased. Therefore, this condition probably will not constitute a significant limitation on use of turbine stator adjustment for this application.

Although the two engines herein have been referred to as the Mach 2.5 and Mach 3.0 engines, the direction of the trend of turbine design variables is probably not so much dependent on the flight Mach number as on the location relative to the break-even point. Instead, the flight Mach number dictates the extent to which the turbine operating conditions must be varied to achieve satisfactory take-off performance.

#### SUMMARY OF RESULTS

The method of engine off-design operation that keeps the compressor at its design point by means of turbine stator adjustment was analytically evaluated as a means of obtaining high engine thrust during take-off from an engine suitable for operation at high supersonic speeds. The following two sets of conditions were considered the engine design conditions:

Flight Mach number	Compressor pressure ratio	Turbine-inlet temperature, °R
2.5	3	2000
3.0	2	3000

The following results were obtained:

1. For the Mach 3.0 engine, the off-design turbine requirements are considerably more severe than for the Mach 2.5 engine.

2. The largest change in stator-exit angle between design-point and take-off operation was encountered with the Mach 3.0 engine, for which the angle was changed from a value of  $34^\circ$  to a value of  $92^\circ$ . The rotor-entrance angle varied by as much as  $51^\circ$  from the design value, the deviation producing negative incidence angles. The rotor-entrance Mach number increases in some cases and decreases in others in comparison with the design values.

3. Because of the low compressor pressure ratio and high turbine-inlet temperature of the Mach 3.0 engine, the design value of engine equivalent weight flow for a given turbine-tip frontal area had to be decreased 16 percent in order that the rotor blade-loading limit would not be exceeded during take-off operation at rated turbine-inlet temperature. Despite this decrease in flow, the take-off thrust for a turbine of a given frontal area increased by 73 percent.

#### CONCLUSIONS

For engine off-design operation with the compressor kept at its design point by means of turbine stator adjustment, the effect of incorporating this adjustment is to make the turbine design sensitive to the particular engine design conditions selected.

For some engine design conditions, the turbine must be conservatively designed for the high-speed flight conditions in order that the turbine can operate satisfactorily for take-off. For such an engine, the engine thrust during take-off and flight at low speeds should be limited to the minimum acceptable values in order to reduce the impairment of the engine design-point performance.

A new concept, the break-even point, can be used to evaluate quickly whether or not a turbine for a given service will approach the blade-loading limit during off-design operation.

Lewis Flight Propulsion Laboratory  
National Advisory Committee for Aeronautics  
Cleveland, Ohio, July 9, 1953

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## APPENDIX A

## VELOCITY DIAGRAM COMPUTATION

## Design-Point Velocity Diagrams

For convenience, the subscript *des* has been omitted from all of the following equations for the design-point velocity diagrams, with the exception of equation (A2).

Rotor exit. - At the design point

$$V_{u,5} = 0 \quad (A1)$$

by assignment. A further specification is made that

$$\left. \begin{aligned} \left( \frac{\rho V_x}{\rho' a'_{cr}} \right)_{5,des} &= 0.562 \frac{A_{cr,5,des}}{A_{cr,5}} \\ \text{or} \\ \left( \frac{\rho V_x}{\rho' a'_{cr}} \right)_{5,des} &= 0.562 \end{aligned} \right\} \quad (A2)$$

whichever is smaller. In reference 8, figure 4, which is a graphic solution of the equation

$$\frac{\rho V_x}{\rho' a'_{cr}} = \left\{ 1 - \frac{k-1}{k+1} \left[ \left( \frac{V_x}{a'_{cr}} \right)^2 + \left( \frac{V_u}{a'_{cr}} \right)^2 \right] \right\}^{\frac{1}{k-1}} \left( \frac{V_x}{a'_{cr}} \right)$$

yields the value of the parameter  $(V_x/a'_{cr})_5$  upon substitution of equations (A1) and (A2).

At the mean radius,

$$\frac{gJ(h_3^* - h_5^*)}{U_m^2} = \frac{gJE}{U_h^2} \left( \frac{r_h}{r_t} \frac{r_t}{r_m} \right)^2 \quad (A3)$$

Evaluation of the factor  $(h_3^* - h_5^*)$  from conventional cycle analysis together with equation (12) permits calculation of  $U_m$ , which is a constant

for each of the two engines considered. With  $T_5^1$  also having been found from cycle analysis, the equation

$$a_{cr}^1 = \sqrt{\left(\frac{2k}{k+1}\right) gRT^1} \quad (A4)$$

can be used in determining  $(U_m/a_{cr}^1)_5$ .

From the velocity diagram of figure 1(b), the following relations are established:

$$V^2 = V_u^2 + V_x^2 \quad (A5)$$

$$W^2 = V_x^2 + (V_u - U)^2 \quad (A6)$$

$$\alpha = \tan^{-1} \frac{V_x}{V_u} \quad (A7)$$

$$\beta = \tan^{-1} \frac{V_x}{V_u - U} \quad (A8)$$

Equation (A6) is used to calculate  $(W_m/a_{cr}^1)_5$ , from which the rotor-exit Mach number  $(W/a)_{5,m}$  is found by

$$\frac{W}{a} = \frac{W}{a_{cr}^1} \frac{a_{cr}^1}{a} \quad (A9)$$

where

$$\frac{a}{a_{cr}^1} = \sqrt{\frac{k+1}{2}} \sqrt{1 - \frac{k-1}{k+1} \left[ \left(\frac{V_x}{a_{cr}^1}\right)^2 + \left(\frac{V_u}{a_{cr}^1}\right)^2 \right]} \quad (A10)$$

Rotor entrance. - By virtue of equation (A1), Euler's work equation can be written

$$\frac{gJ(h_3^1 - h_5^1)}{U_m^2} = \frac{V_{u,4,m}}{U_m} \quad (A11)$$

from which  $V_{u,4,m}$  can be calculated. Since  $T_4^* = T_3^*$ , equation (A4) is used to compute  $(V_{u,m}/a'_{cr})_4$ .

If it is assumed that all the turbine loss occurs in the rotor and none in the stator,

$$\left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_4 = \left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_5 \left(\frac{p_5^*}{p_3^*}\right) \sqrt{\frac{T_3^*}{T_5^*}} \quad (A12)$$

if the annular areas  $A_4$  and  $A_5$  are equal. Recourse to figure 4 of reference 8 yields the parameter  $(V_x/a'_{cr})_4$ , since  $(V_{u,m}/a'_{cr})_4$  and  $(\rho V_x/\rho' a'_{cr})_4$  are known. The entrance velocity diagram is completed by use of equations (A4) to (A8).

Stator-exit Mach number  $(V/a)_{4,m}$  is computed from the relation

$$\frac{V}{a} = \frac{V}{a'_{cr}} \frac{a'_{cr}}{a} \quad (A9a)$$

and equation (A10). Similarly, rotor-entrance Mach number  $(W/a)_{4,m}$  is calculated from equations (A9) and (A10).

The velocity ratio  $(W_5/W_4)_m$  is calculated from the corresponding Mach numbers, as follows:

$$\frac{W_5}{W_4} = \sqrt{\frac{T_5^*}{T_3^*}} \sqrt{\frac{1 - \frac{k-1}{k+1} \left(\frac{V}{a'_{cr}}\right)_5^2 \left(\frac{W}{a}\right)_5^2}{1 - \frac{k-1}{k+1} \left(\frac{V}{a'_{cr}}\right)_4^2 \left(\frac{W}{a}\right)_4^2}} \quad (A13)$$

#### Take-Off Velocity Diagrams

Rotor exit. - The assumptions used in calculation of the take-off velocity diagrams are

$$\left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_5 = \left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{5,des} \left(\frac{A_{cr}}{A_{cr,des}}\right)_5 \quad (A14)$$

and

$$\beta_{5,m} = \beta_{5,m,des} \quad (A15)$$

Equation (A8) can be rearranged

$$\left(\frac{V_x}{a'_{cr}}\right)_5 = \left(\frac{V_{u,m}}{a'_{cr}}\right)_5 \tan \beta_{5,m} - \left(\frac{U_m}{a'_{cr}}\right)_5 \tan \beta_{5,m} \quad (A16)$$

in which

$$U_m = U_{m,eq}$$

and  $a'_{cr,5}$  is calculated from the value of  $T_5^*$  found by cycle analysis. Solution of equation (A16) for  $(V_x/a'_{cr})_5$  and  $(V_{u,m}/a'_{cr})_5$  can be effected by a method of successive approximations in conjunction with figure 4 of reference 8. Convergence having been obtained, the rotor-exit velocity diagram is completed by use of equations (A5) and (A6), and the Mach number is calculated by equations (A9) and (A10).

Rotor entrance. - Since the work at take-off is assigned equal to that at the equivalent point,  $V_{u,4,m}$  is calculated from the relation

$$V_{u,4} - V_{u,5} = (V_{u,4} - V_{u,5})_{eq} \quad (A17)$$

and equation (A1). Equation (A4) yields  $(V_{u,m}/a'_{cr})_4$ , and equation (A12) yields  $(\rho V_x / \rho' a'_{cr})_4$ , so that entry into figure 4 of reference 8 gives  $(V_x/a'_{cr})_4$ . Equations (A5) to (A8) suffice to complete the entrance velocity diagram. Stator-exit Mach number is calculated from equations (A4), (A9a), and (A10); rotor-entrance Mach number is computed by equations (A4), (A9), and (A10). Equation (A13) is used for determining the velocity ratio.



## APPENDIX B

## EFFECT OF APPROXIMATION IN VELOCITY DIAGRAM COMPUTATIONS

In the calculations for the velocity diagrams the value of the parameter  $(\rho V_x / \rho' a'_{cr})_5$  was arbitrarily chosen as 0.562 for simplicity. This is 10.5 percent less than the choking value of 0.628. Reference 5, however, shows that it would be preferable to assign a value of 0.7 for the exit axial Mach number  $(V_x/a)_5$ . Assignment of a value for this latter parameter complicates the calculations considerably. To investigate the error introduced by using  $(\rho V_x / \rho' a'_{cr})_5 = 0.562$  instead of  $(V_x/a)_5 = 0.7$ , one set of calculations was made with the latter specification for the Mach 3.0 engine at a turbine-inlet temperature for take-off of 3000° R; flow conditions at this point are most extreme. Table I presents a comparison of the velocity diagrams computed at this point for the two modes of calculation. This table reveals that although the numerical values have changed for the diagram obtained by assuming  $(V_x/a)_5 = 0.7$ , the over-all result is the same; that is, large changes between the equivalent and the take-off diagrams are still observed.

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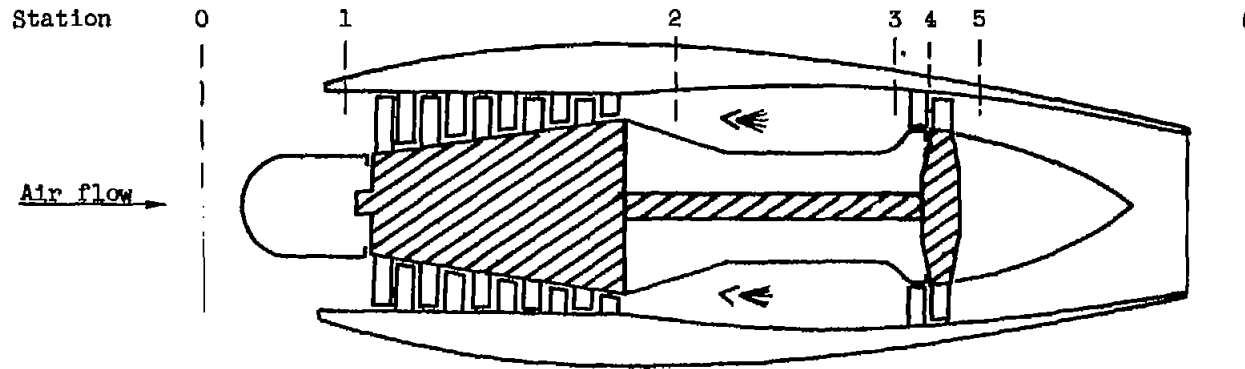
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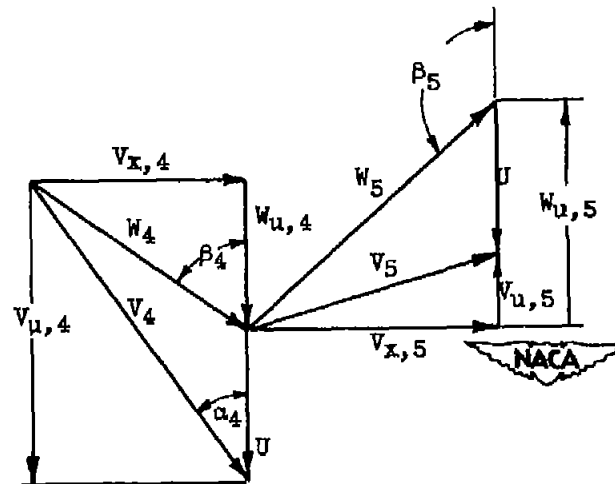
TABLE I. - VELOCITY DIAGRAMS COMPUTED FROM  $(\rho V_x / \rho' a'_{cr})_5 = 0.562$   
 COMPARED WITH THOSE COMPUTED FROM  $(V_x / a)_5 = 0.7$  FOR MACH 3.0  
 ENGINE WITH 3000° R TAKE-OFF TURBINE-INLET TEMPERATURE

	$(\rho V_x / \rho' a'_{cr})_5 = 0.562$		$(V_x / a)_5 = 0.7$	
	Design	Take-off	Design	Take-off
$(\rho V_x / \rho' a'_{cr})_4$	0.325	0.473	0.306	0.446
$(V_x / a'_{cr})_4$	.419	.539	.390	.498
$(V_u / a'_{cr})_4$	.620	-.019	.620	.048
$(V / a'_{cr})_4$	.746	.540	.731	.501
$(V / a)_4$	.722	.513	.707	.475
$(W / a)_4$	.428	.610	.401	.542
$\alpha_4$	34.04°	91.98°	32.16°	84.50°
$\beta_4$	71.41°	122.84°	70.13°	119.44°
$(\rho V_x / \rho' a'_{cr})_5$	0.472	0.562	0.444	0.529
$(V_x / a'_{cr})_5$	.534	.912	.494	.713
$(V_u / a'_{cr})_5$	0	-.453	0	-.385
$(V / a'_{cr})_5$	.534	1.018	.494	.809
$(W / a)_5$	.695	1.209	.666	.988
$W_5 / W_4$	1.588	1.845	1.626	1.740
$\beta_5$	133.07°	130.84°	134.70°	134.70°
$V_{u,5}^2 / 2gJF$	0	0.705	0	0.509

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(a) Cross section of turbojet engine showing location of numerical stations.



(b) Typical velocity diagram.

Figure 1. - Schematic illustrations showing notation.

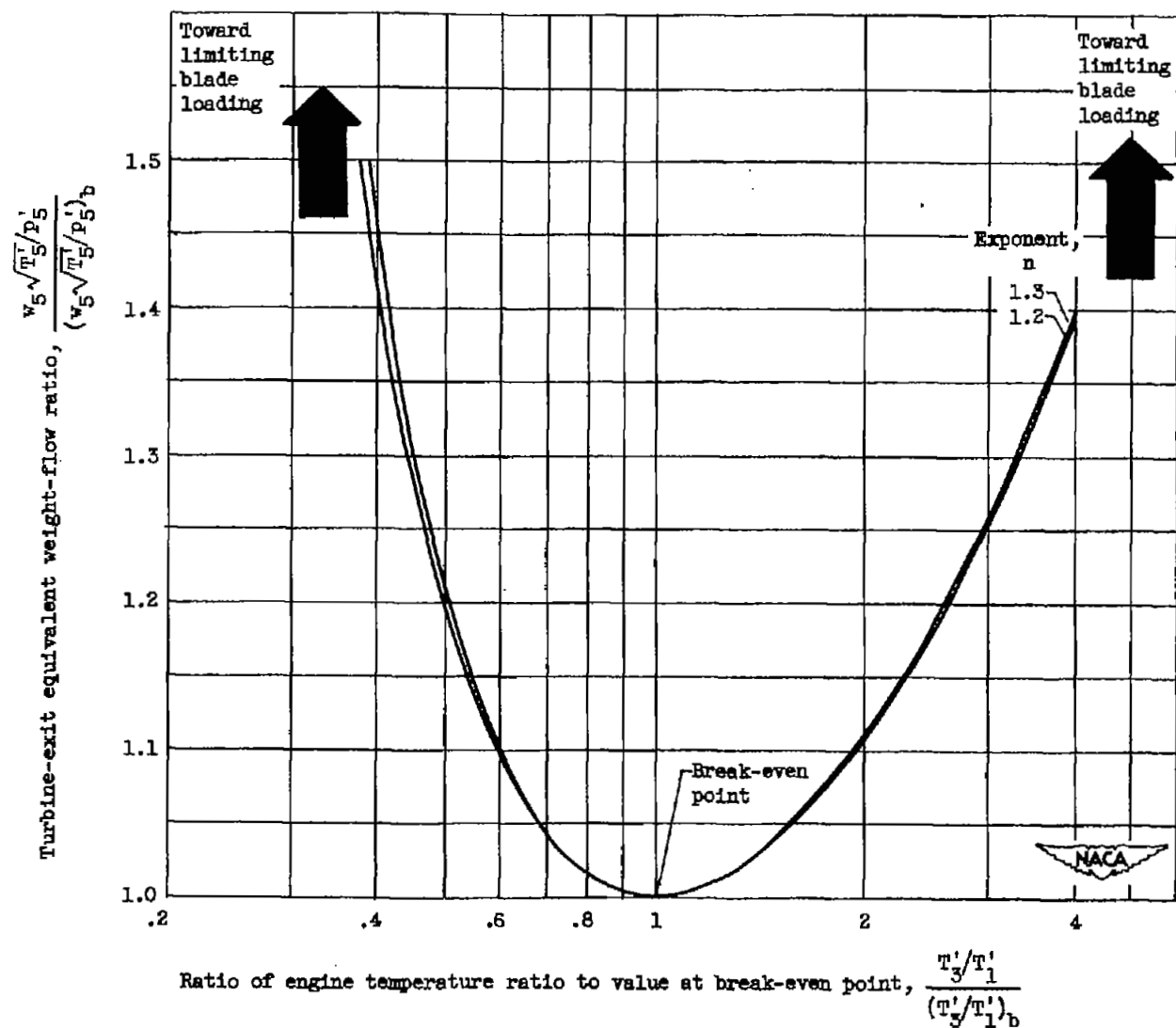
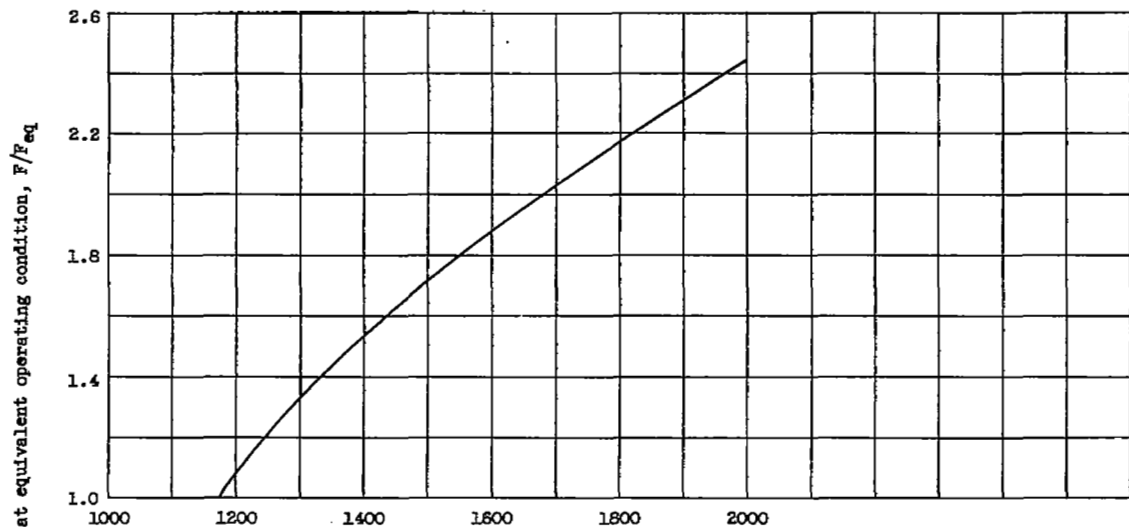
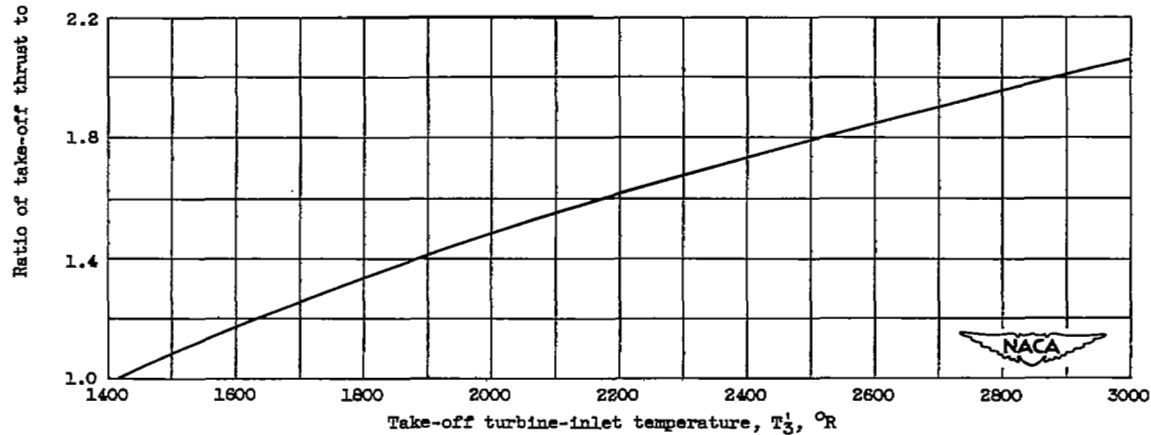


Figure 2. - Effect of engine temperature ratio on turbine-exit equivalent weight flow.



(a) Mach 2.5 engine.



(b) Mach 3.0 engine.

Figure 3. - Variation of take-off thrust with take-off turbine-inlet temperature.

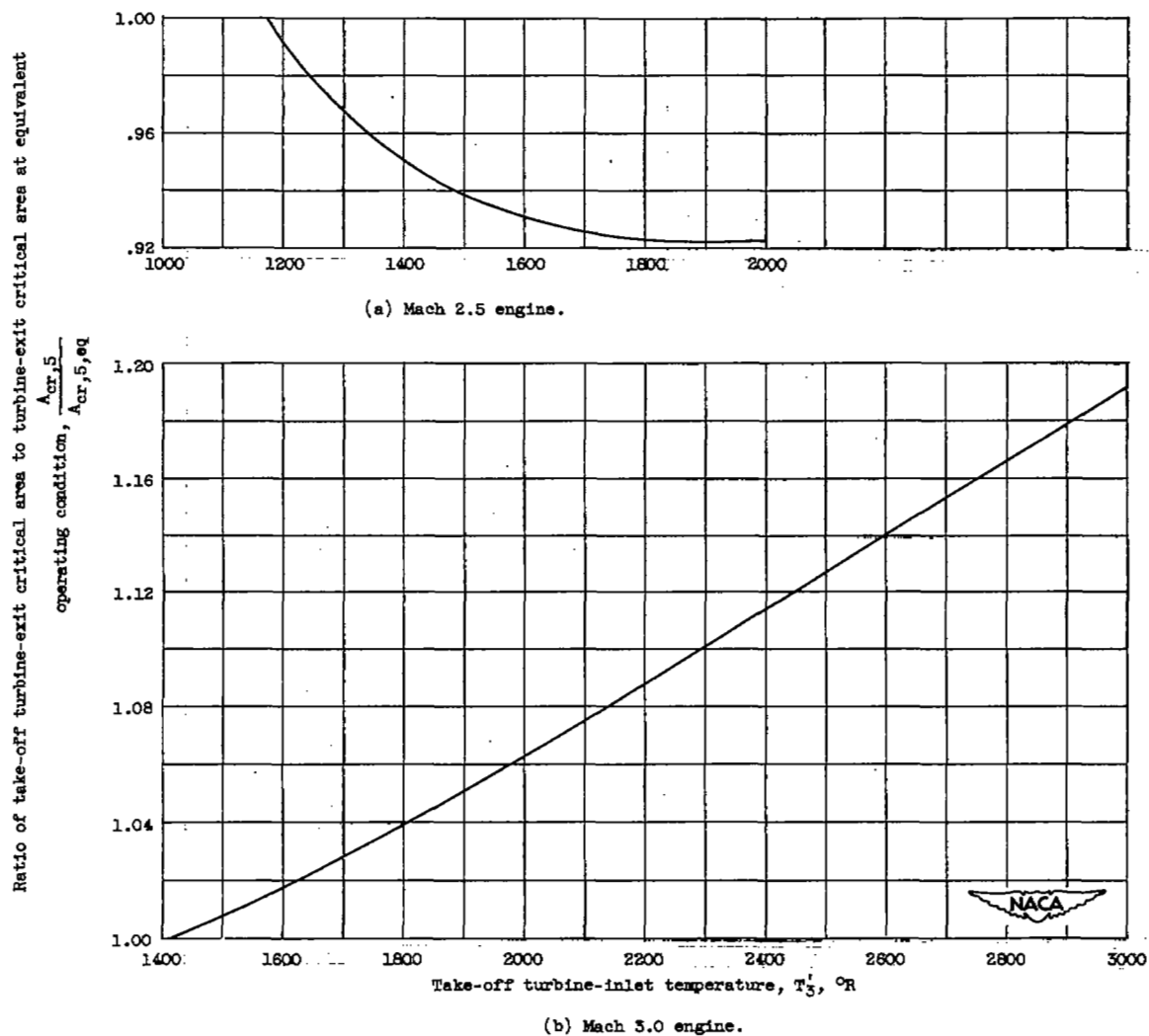


Figure 4. - Variation of take-off turbine-exit critical area with take-off turbine-inlet temperature.

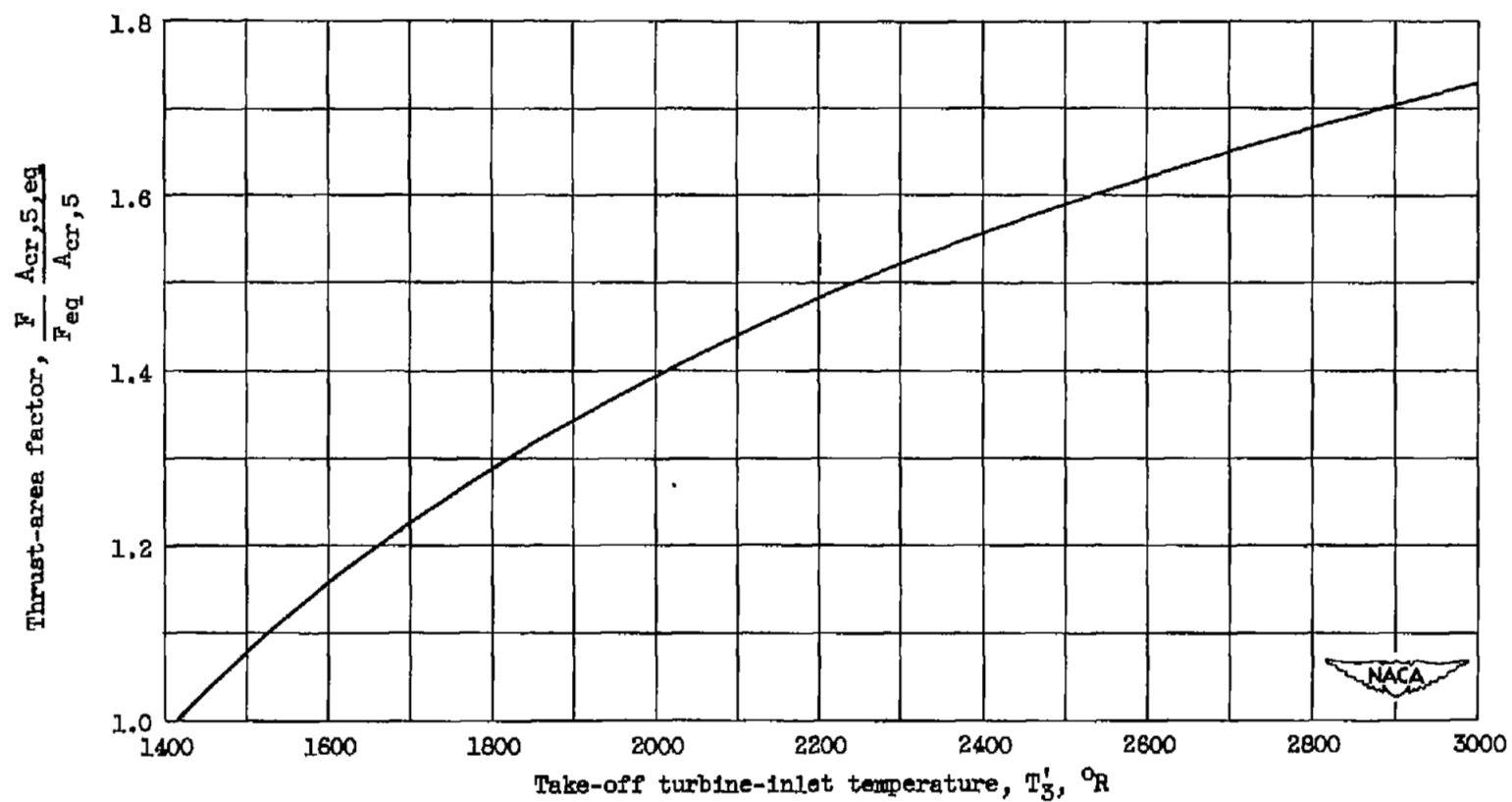


Figure 5. - Variation of thrust-area factor with take-off turbine-inlet temperature for Mach 3.0 engine.



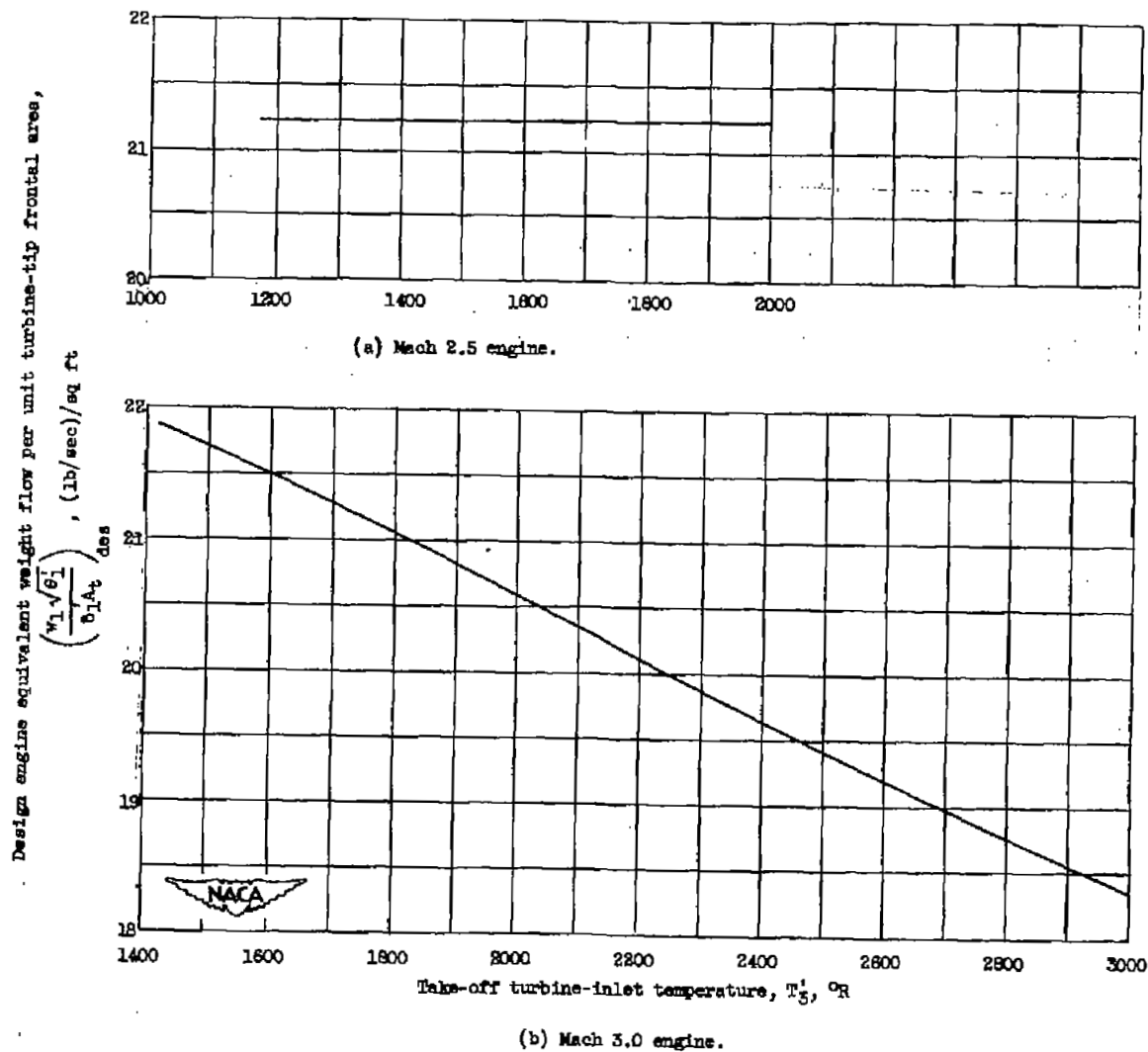
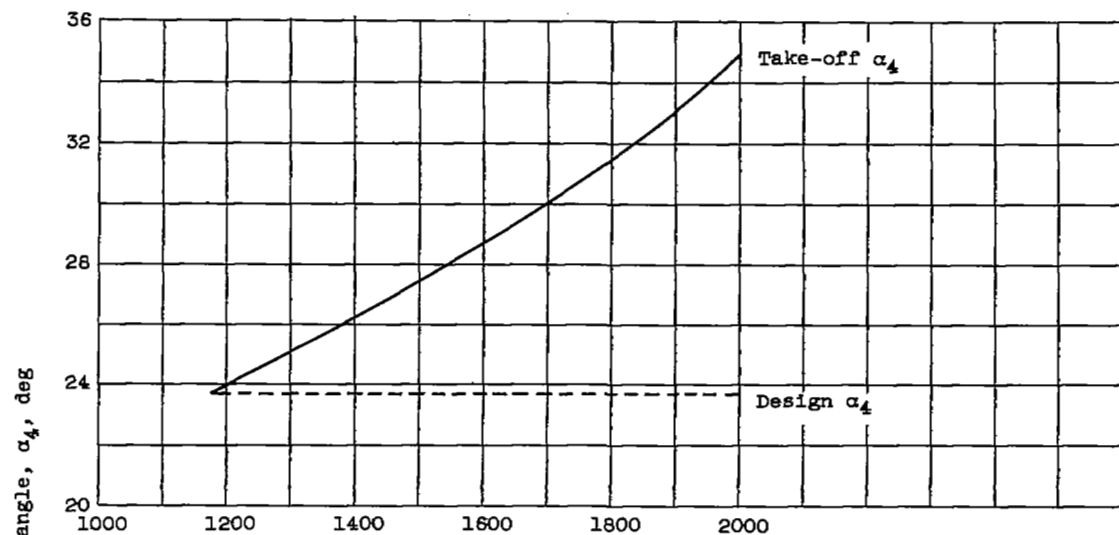


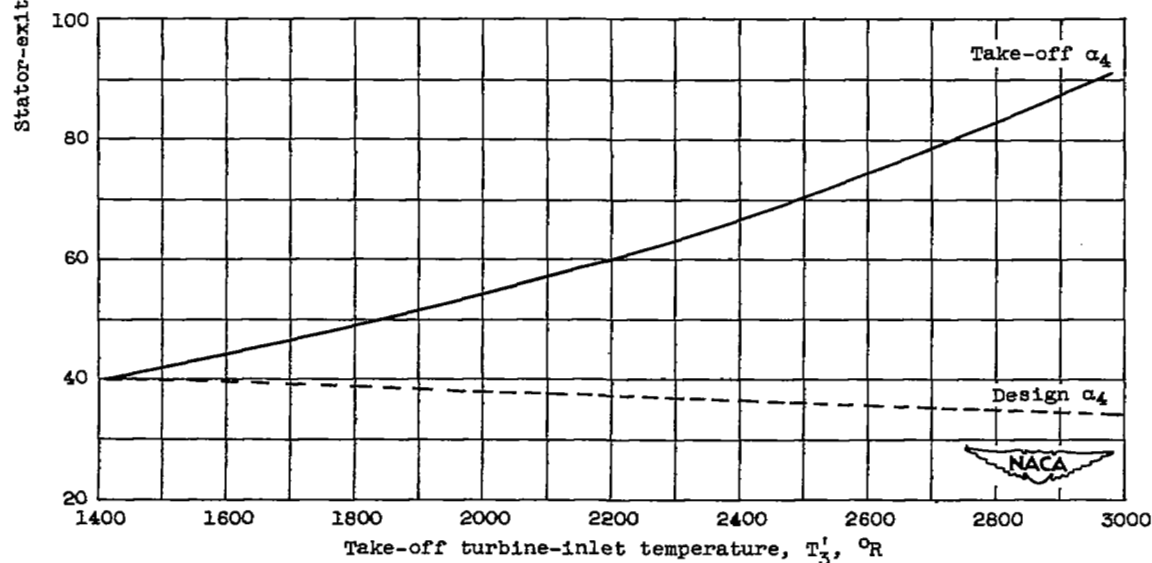
Figure 6. - Variation of design engine equivalent weight flow per unit turbine-tip frontal area with take-off turbine-inlet temperature.

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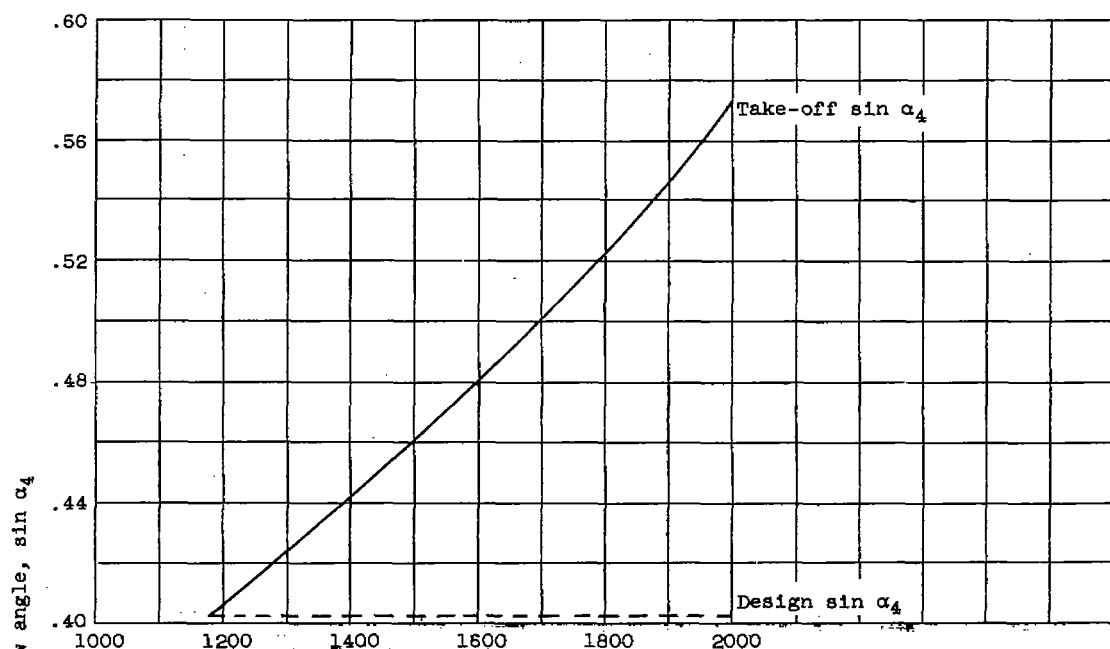


(a) Mach 2.5 engine.

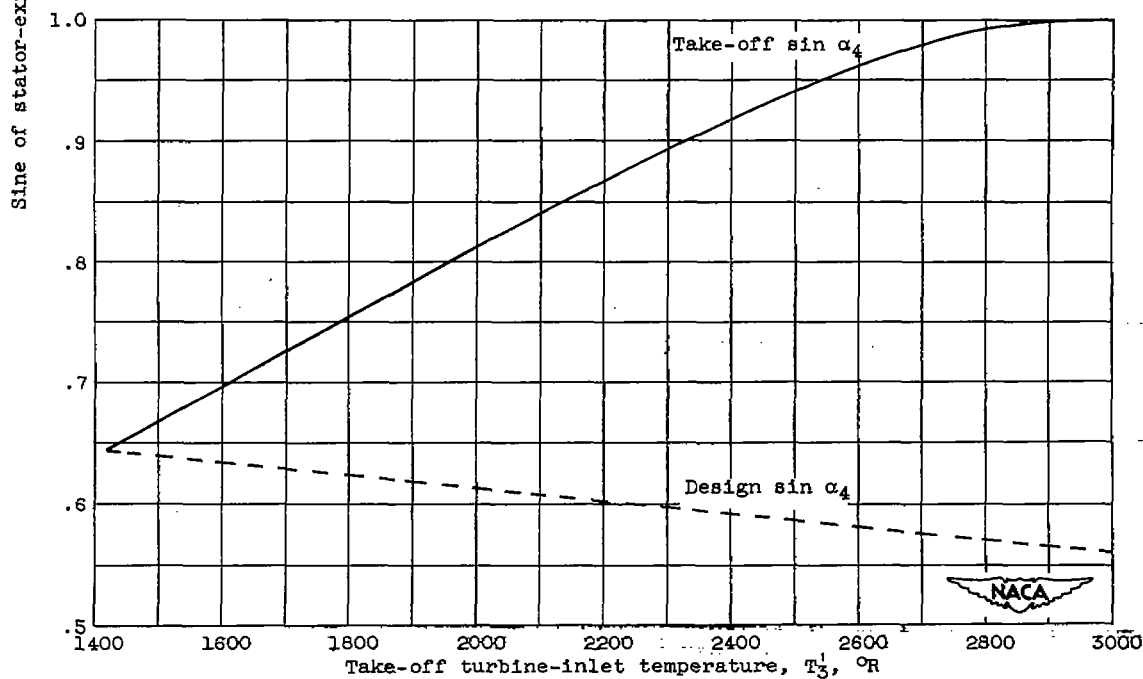


(b) Mach 3.0 engine.

Figure 7. - Variation of take-off stator-exit flow angle with take-off turbine-inlet temperature, and comparison with design stator-exit flow angle.



(a) Mach 2.5 engine.



(b) Mach 3.0 engine.

Figure 8. - Variation of take-off stator-exit flow area with take-off turbine-inlet temperature, and comparison with design stator-exit flow area.

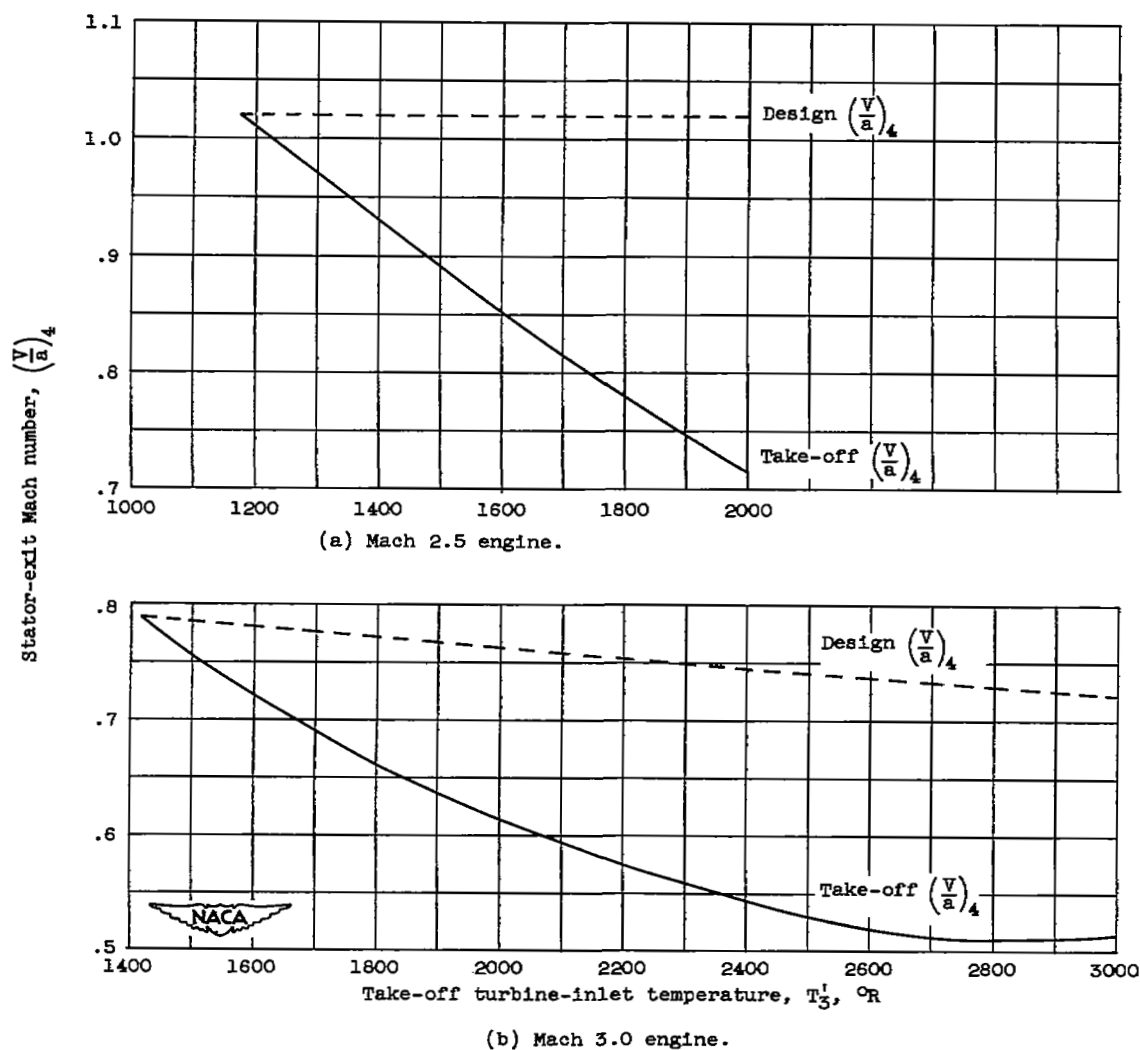


Figure 9. - Variation of take-off stator-exit Mach number with take-off turbine-inlet temperature, and comparison with design stator-exit Mach number.

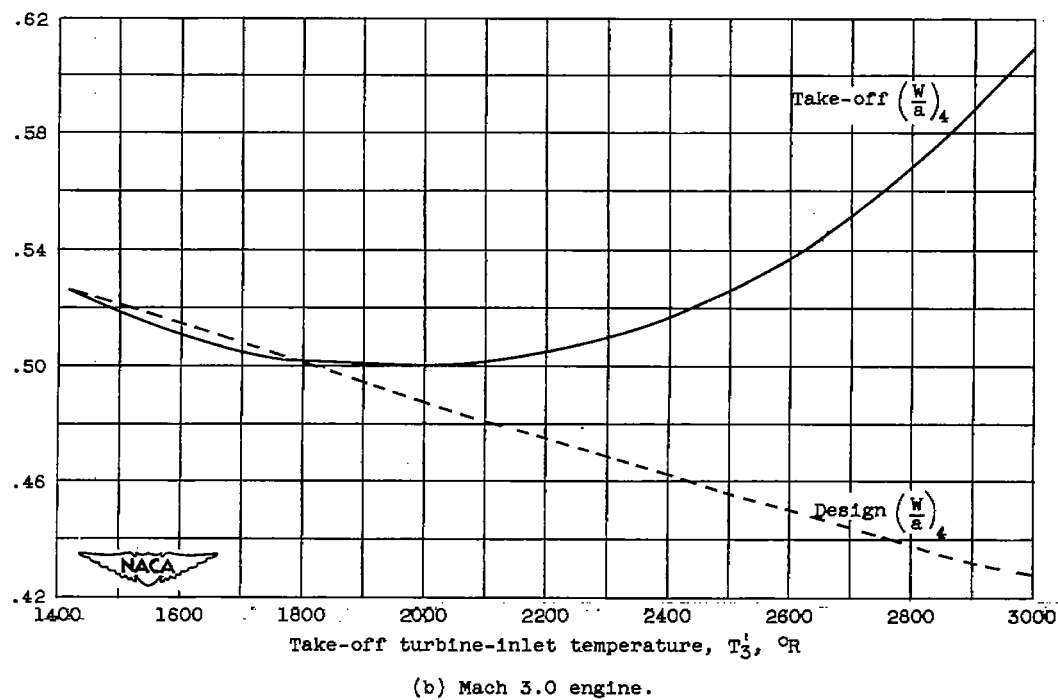
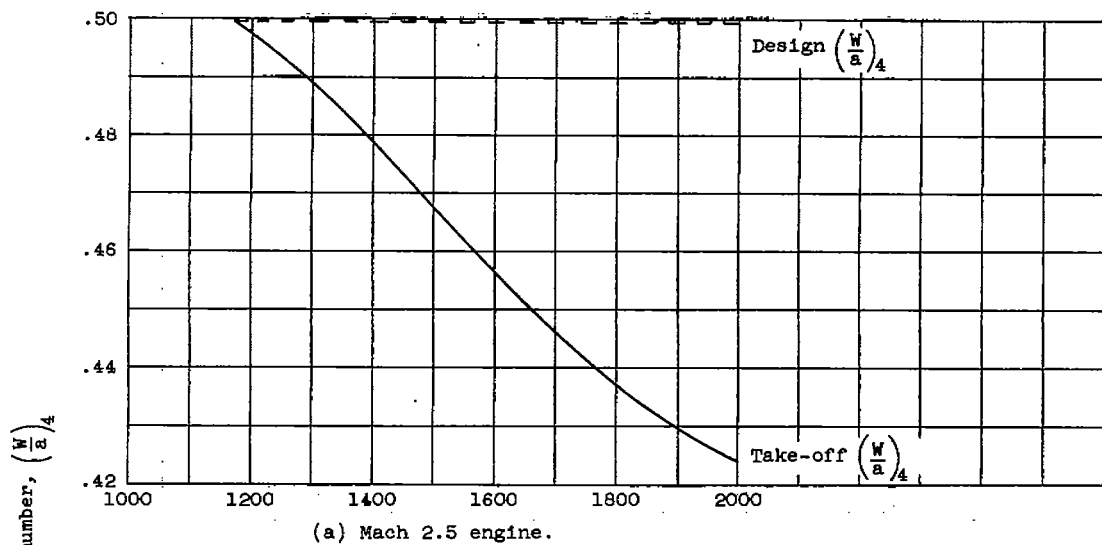
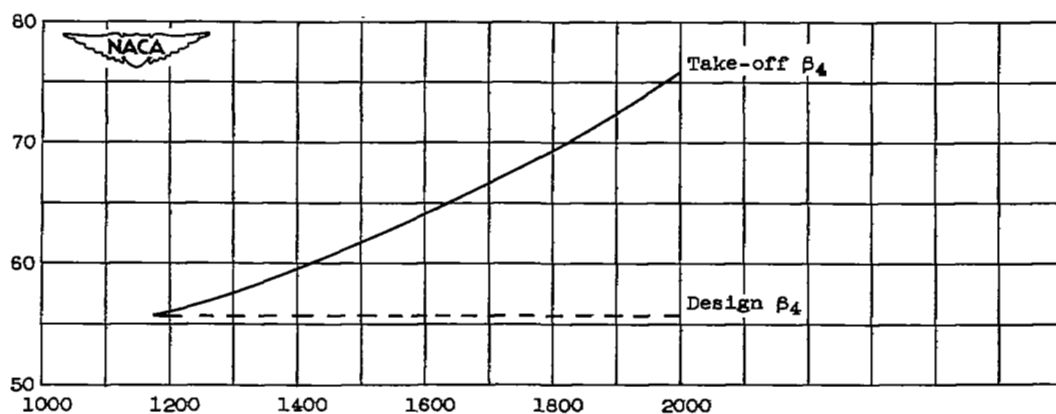
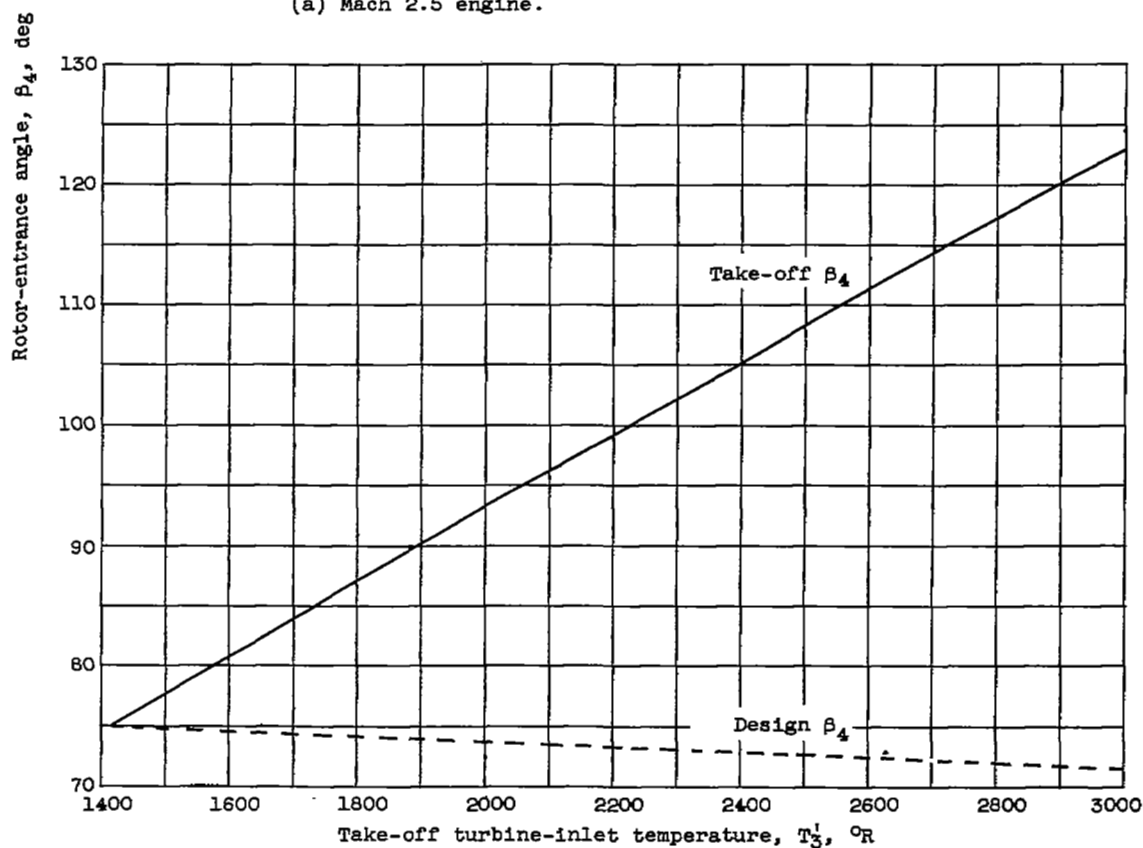


Figure 10. - Variation of take-off rotor-entrance Mach number with take-off turbine-inlet temperature, and comparison with design rotor-entrance Mach number.

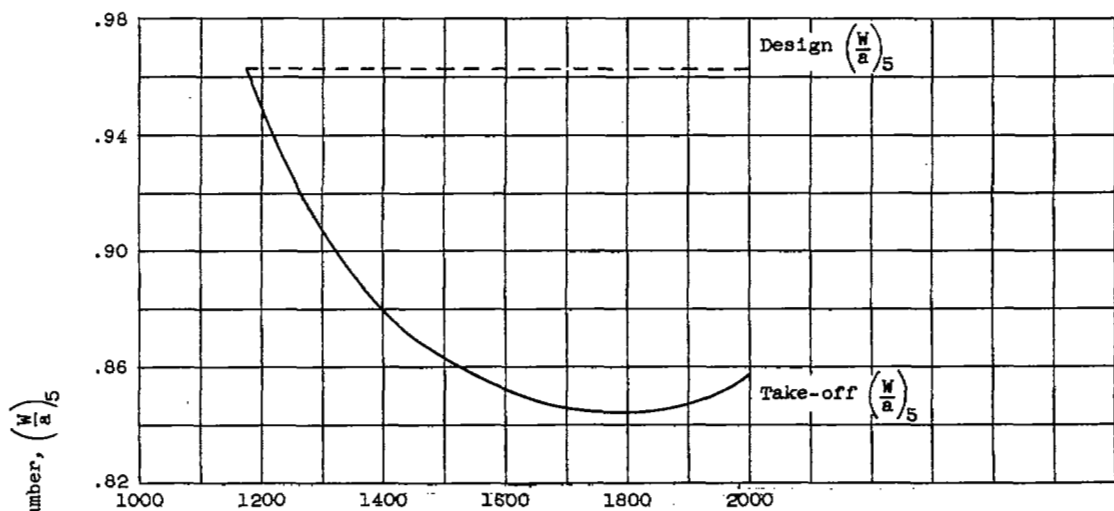


(a) Mach 2.5 engine.

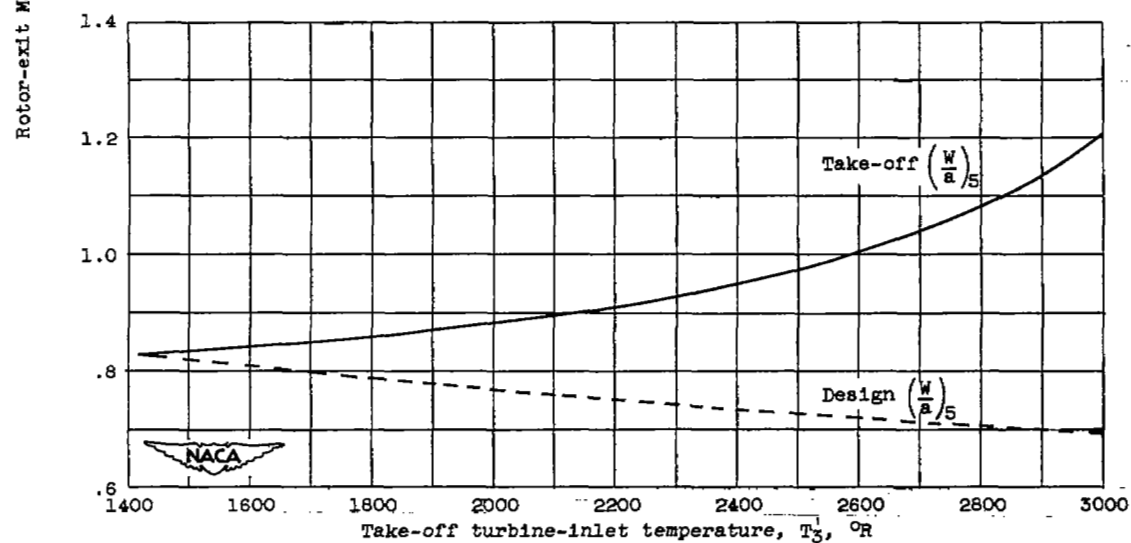


(b) Mach 3.0 engine.

Figure 11. - Variation of take-off rotor-entrance angle with take-off turbine-inlet temperature, and comparison with design rotor-entrance angle.

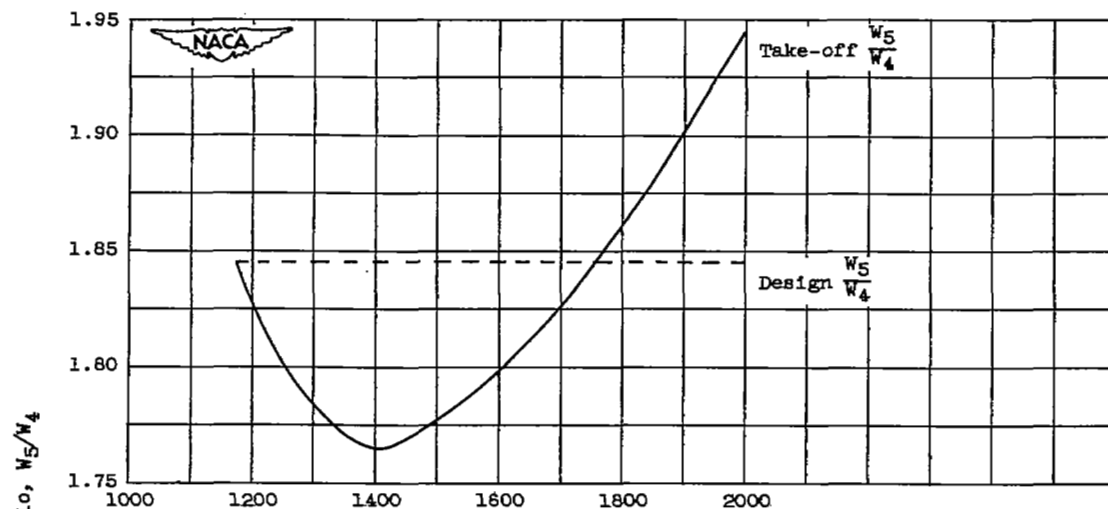


(a) Mach 2.5 engine.

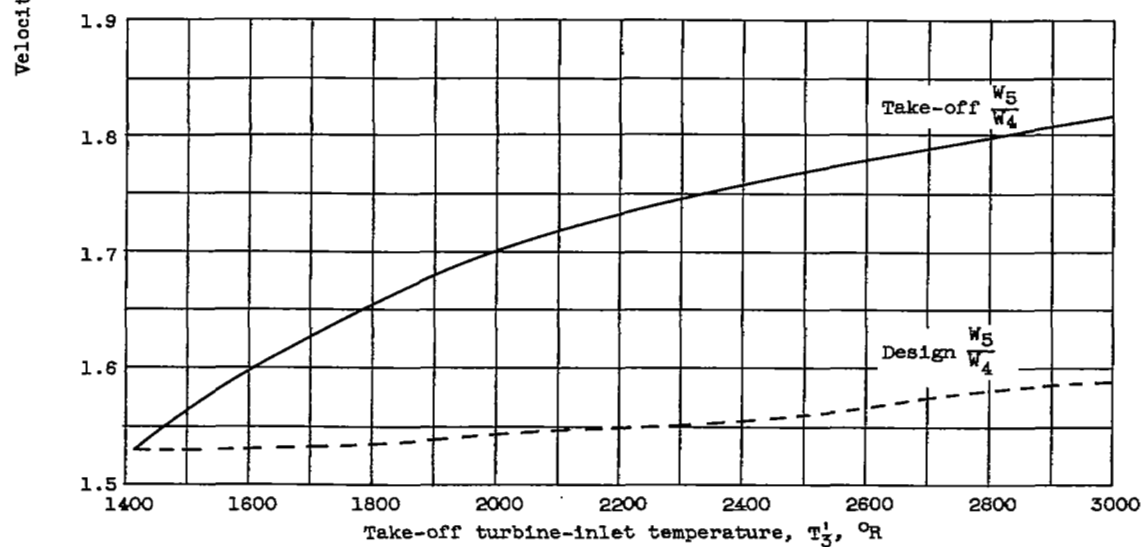


(b) Mach 3.0 engine.

Figure 12. - Variation of take-off rotor-exit Mach number with take-off turbine-inlet temperature, and comparison with design rotor-exit Mach number.



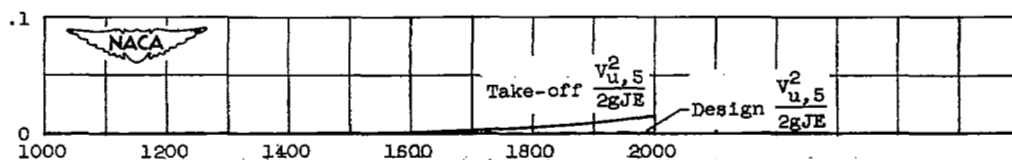
(a) Mach 2.5 engine.



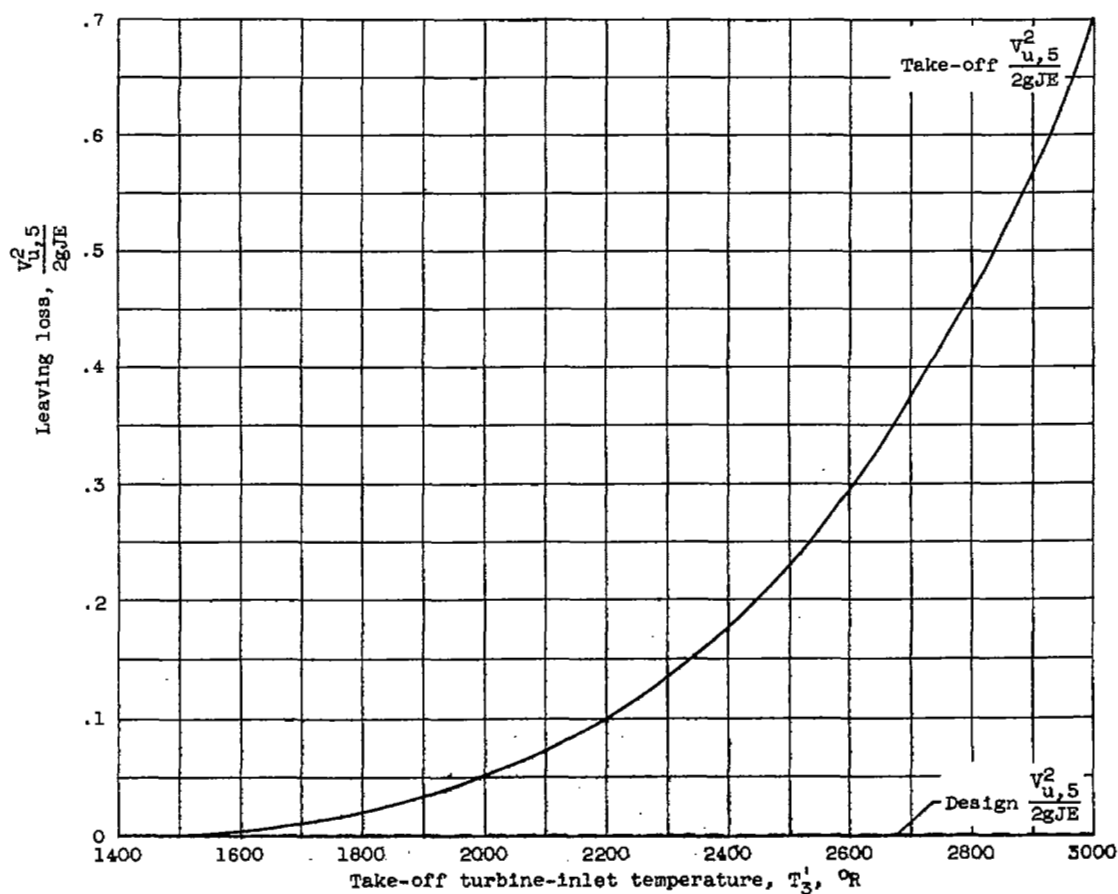
(b) Mach 3.0 engine.

Figure 13. - Variation of take-off value of ratio of rotor-exit to rotor-entrance relative velocities with take-off turbine-inlet temperature, and comparison with design values.





(a) Mach 2.5 engine.



(b) Mach 3.0 engine.

Figure 14. - Variation of take-off value of leaving loss with take-off turbine-inlet temperature, and comparison with design value.

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